## 7.2 ~ Equivalent Quadratic Forms

*Daily Objectives:*

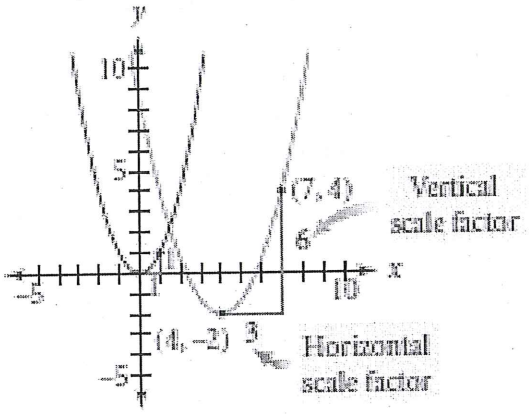
1. Understand the correspondence between the zeros of a polynomial function and the roots of an equation.
2. Use the zero-product property to find the roots of equations.
3. Comprehend the relationships among the general form, the factored form, and the vertex form of a quadratic equation.
4. Relate the vertex form of a quadratic equation to the parent functions $y = x^2$.

A quadratic function in the form $\frac{y-k}{b} = \left(\frac{x-h}{b}\right)^2$ or $y = b\left(\frac{x-h}{a}\right)^2 + k$.

The above equations identifies the location of the vertex ($h$, $k$), and the horizontal and vertical scale factors, $a$ and $b$.

Given the equation $y = 6\left(\frac{x-4}{3}\right)^2 - 2$ which represents the equation of the parabola below.

Rewrite this equation with a combined horizontal and scale factor.




This new form $y = a(x-h)^2 + k$ represents **vertex form**.

$y = a(x-4)^2 - 2$

$4 = a(7-4)^2 - 2$

$4 = a(3)^2 - 2$

$4 = 9a - 2$

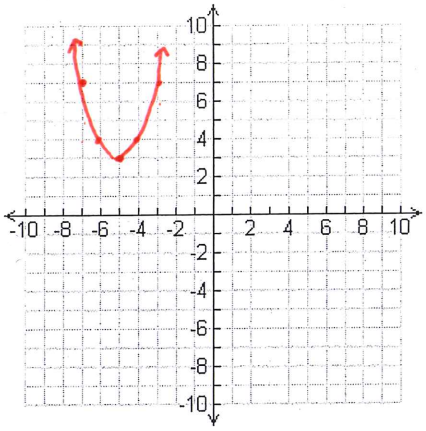
$+2 \quad +2$

$\frac{6}{9} = \frac{9a}{9}$

$\frac{2}{3} = a$

$y = \frac{2}{3}(x-4)^2 - 2$

**Example 1:** Find the vertex of the quadratic represented by the equation $y=(x+5)^2+3$. Then graph the parabola below.




**Zero-Product Property**

For all real numbers $a$ and $b$, if $ab = 0$, then $a = 0$, or $b = 0$, or $a = 0$ and $b = 0$.

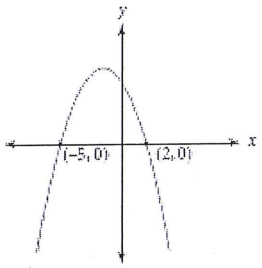
*Fill in the blanks below to satisfy each equation:*

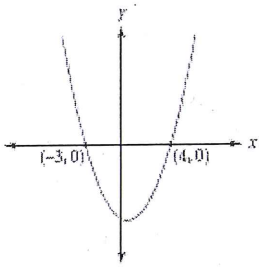
$\underline{0}\cdot 18.9=0$ $\qquad$ $3(\underline{4}-4)(\underline{-9}+9)=0$

**Zeros:**

*Identify the zeros of the graph below:*

$x=-5$
$x=2$




$x=-3$
$x=4$

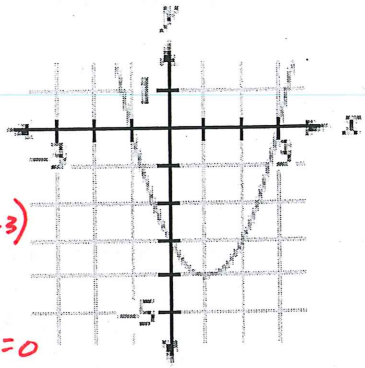
The zeros of a function will be the x-values that make $y$ equal to zero.

**Example 2:** Find the zeros of the function $y = -1.4(x-5.6)(x+3.1)$.

$x-5.6=0$ $\quad x=5.6$ $\qquad$ $x+3.1=0$ $\quad x=-3.1$

The quadratic function written in Example 2 is written in *factored form*.

**Example 3:** Consider the parabola below:



a. Write an equation of the parabola in vertex form.

$y=(x-1)^2-4$

b. Write an equation of the parabola in factored form.
c. Show that both equations are equivalent.

$y=(x+1)(x-3)$

$0=(x+1)(x-3)$

$x+1=0 \quad x-3=0$

$-1 \quad -1 \quad +3 \quad +3$

$x=-1 \quad x=3$

$(x-1)^2-4$

$(x-1)(x-1)-4$

$x^2-2x+1-4$

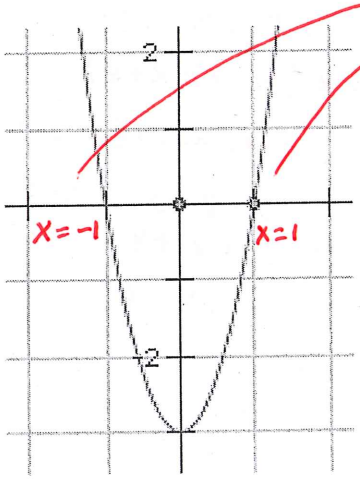
$x^2-2x-3$

$(x+1)(x-3)$

$x^2-3x+1x-3$

$x^2-2x-3$

**Example 3:** Given the parabola below, write its equations in factored and vertex form.




$y=(x-1)(x+1)$ $\qquad$ $y=x^2-3$

To find the scale factor $a$, substitute a point from your graph into the equation. Then solve for $a$.

**Three Forms of a Quadratic Function**

| | |
|---|---|
| General form | $y = ax^2 + bx + c$ |
| Vertex form | $y = a(x-h)^2 + k$ |
| Factored form | $y = a(x-r_1)(x-r_2)$ |

**Example 4:** Write each quadratic formula below in general form:

a. $y=(x+8)(x-1)$

$y = x^2+7x-8$

b. $y=(x+4)^2-10$

$y = x^2+8x+16-10$

$y = x^2+8x+6$

c. $y=(x-3)(x-4)$

$y = x^2-7x+12$

d. $y=(x-5)^2-8$

$y = x^2-10x+25-8$

$y = x^2-10x+17$

e. $y=3(x+2)^2-8$

$y = 3(x^2+4x+4)-8$

$= 3x^2+12x+12-8$

$y = 3x^2+12x+4$

**Example 5:** Write each general form in factored form: and find the zeroes!

a. $y=x^2-10x+24$

$y = (x-6)(x-4)$

$0 = (x-6)(x-4)$

$x-6=0 \quad x-4=0$

$+6 \;+6 \quad +4 \;+4$

$x=6 \quad x=4$

b. $y=x^2+8x-20$

$y = (x+10)(x-2)$

$0 = (x+10)(x-2)$

$x+10=0 \quad x-2=0$

$-10 \;-10 \quad +2 \;+2$

$x=-10 \quad x=2$

c. $y=2x^2-6x-80$

$2(x^2-3x-40)$

$y = 2(x-8)(x+5)$

$0 = 2(x-8)(x+5)$

$x-8=0 \quad x+5=0$

$+8 \;+8 \quad -5 \;-5$

$x=8 \quad x=-5$

Example 6: Find the equation of a graph with x-intercepts at (4,0) and (-3,0) if the graph passes through the point (6,-36).

$y = a(x-4)(x+3)$

$-36 = a(6-4)(6+3)$

$-36 = a \cdot 2 \cdot 9$

$\frac{-36}{18} = \frac{18a}{18}$

$-2 = a$

$y = -2(x-4)(x+3)$